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Leading Power Corrections in QCD: From Renormalons to Phenomenology

R. Akhoury and **V.I. Zakharov**

The Randall Laboratory of Physics
University of Michigan
Ann Arbor MI 48109

Abstract

We consider $1/Q$ corrections to hard processes in QCD where Q is a large mass scale, concentrating on shape variables in e^+e^- annihilation. While the evidence for such corrections can be and has been established by means of the renormalon technique, theory can be confronted with experiment only after clarifying the properties of the corresponding non-perturbative contribution. We list predictions based on the universality of the $1/Q$ terms, and compare them with the existing data. We also identify the scale of the non-perturbative contributions in terms of jet masses.

1 Introduction

Perturbative QCD constitutes a well defined framework for understanding hard processes, i.e. processes characterized by a large mass scale Q (see for example [1]). While the zeroth order approximation is provided usually by the parton model and is well defined, the radiative corrections can bring logarithmic factors which depend on the infrared cut off and destabilize the theoretical predictions. To avoid this problem, one concentrates usually on a set of infrared safe quantities which are protected against such contributions. There are powerful general theorems on infrared stability of theoretical predictions based on factorization of short and large distances, for a review see [2]. In this note we will consider shape variables in e^+e^- annihilation which are infrared safe. As an example one may keep in mind the thrust T defined as

$$T \;=\; max_{\mathbf{n}}\frac{\Sigma(\mathbf{p}_i\mathbf{n})}{\Sigma|\mathbf{p}_i|} \tag{1}$$

where $\mathbf{p}_i$ are the momenta of the particles produced while $\mathbf{n}$ is a unit vector.

While perturbative QCD allows us to calculate thrust as a series in a small expansion parameter $\alpha_s(Q^2)$ [3]:

$$\langle 1-T\rangle \;=\; 0.335\alpha_s(Q^2)+1.02\alpha_s^2(Q^2)+... \quad , \tag{2}$$

analysis of the experimental data at least at moderate Q^2 indicates also the presence of $1/Q$ corrections [4, 5]. Theoretically, such corrections are exponentially small,

$$1/Q \;\sim\; exp(-b_0/2\alpha_s(Q^2)) \;,$$

where b_0 is the first coefficient in the β-function, and are clearly beyond the reach of a purely perturbative approach. The only consisitent way to deal with the power corrections [6] is provided by the operator product expansion (OPE). However, this technique applies only to a very selective set of physical quantities like the total cross section.

Nevertheless, most a recently QCD-based phenomenology of the $1/Q$ corrections has been emerging [7, 8, 9, 10, 11, 12]. It is worth emphasizing that the very existence of the $1/Q$ corrections can be readily understood. Indeed, consider the emission of a soft gluon with

energy $\omega \sim \Lambda_{QCD}$. The corresponding contribution to the thrust is of order:

$$1 - T_{soft} \;\sim\; \int_0^{\Lambda_{QCD}} \frac{\omega}{Q}\frac{d\omega}{\omega}\alpha_s(\Lambda_{QCD}^2) \;\sim\; \frac{\Lambda_{QCD}}{Q} \tag{3}$$

where the first factor in the integrand comes from the definition of the thrust, $d\omega/\omega$ is the standard factor for emission of a soft gluon, and the running coupling $\alpha_s(\Lambda^2_{QCD})$ is of order unit for a soft gluon. Although T_{soft} cannot be calculated reliably in perturbation theory, the presence of the $1/Q$ corrections is obvious. In reality, the existence of the $1/Q$ corrections has been established [7, 13, 14, 8] not via such simple estimates but rather by means of infrared renormalons [15]. Renormalons, which are a particular set of perturbative graphs, allow us to clarify the convergence properties of the expansions like (2) and estimate their uncertainty as powers of $1/Q$.

While the conclusion on the very existence of $1/Q$ corrections seems to be guaranteed, to develop a phenomenology of such corrections one needs means to relate them in various processes and to fix the overall scale. These are the issues central to the present note, which is considered to be a continuation of the analysis started in [11].

It might be worth emphasizing that renormalons *per se* cannot be a basis for such a phenomenology. Indeed, to apply renormalons literally one has to calculate all the terms in the expansion (2) until they start to rise in the fashion prescribed by the renormalons. The power-like terms appear then as an uncertainty of the asymptotic expansion and are dependent on the procedure for subtracting the perturbative contributions. The ambiguities of such a procedure are spelled out in Ref [16] and it has never been tried so far.

The attempts to develop the phenomenology of $1/Q$ terms are based therefore on a mixture of theoretical inputs and heuristic arguments. The first estimation of $1/Q$ terms in shape variables [8] relied on the version of the renormalon technique which replaces the renormalons by terms non analytical in the gluon mass squared, μ^2 [13, 17]. This procedure can be thought of only in the lowest order in α_s. This is also true for the modification of the technique which assumes freezing of the running coupling at some scale [10]. The analysis of the data indicated that experimentally $1/Q$ terms are proportional to the μ/Q corrections found theoretically [8]. Another line of development [7, 9] is to evaluate the renormalon contributions to a given process to all orders and to develop an operator analysis based on

an effective theory arising in the eikonal approximation [2].

In our previous paper [11] we argued that terms of all orders in the large coupling $\alpha_s(\Lambda^2_{QCD})$ factorize in the $1/Q$ corrections to various shape variables into a universal factor. We argued furthermore that non-perturbative contributions share this property of factorization since they are also associated with distances much larger then $1/Q$. As a result, one can both substantiate the relations among the $1/Q$ terms in the shape variables [8] and try to develop a machinery for deriving furthur experimental consequences from QCD. Similar relations for soft perturbative parts are derived in Ref [12]; no experimental consequences are claimed however because of the reservations for unknown non-perturbative effects.

In this paper we first consider the renormalon technique versus the general operator product expansion in case of the total cross section when both approaches apply. We check on this example the hypotheses concerning the non-perturbative terms made in [11]. We give a list of experimental consequences which follow from the universality of $1/Q$ corrections, non-perturbative contributions including. Partly the predictions (or their variations) were discussed earlier in [8, 10, 11]. Finally, we identify the overall scale of the $1/Q$ corrections in terms of the non-perturbative contribution to jet masses which arise in the Feynman-Field type models. The considerations of this paper are restricted to the kinematic configuration where the dominant nonperturbative corrections come from the vicinity of the 2-jet limit in e^+e- annihilation.

The outline of the paper is as follows. In sect. 2 we review the renormalon technique in case of the total cross section. In sect. 3 we turn to an analysis of the $1/Q$ corrections to certain infrared safe observables in e^+e^- annihilation into jets and list predictions which follow from the universality of the $1/Q$ corrections. In sect. 5 we compare the predictions with existing experimental data. In sect. 6 we establish the correspondence of the renormalon based picture for the power corrections with the phenomenological decription of the non-perturbative effects in jet physics a la Feynman-Field.

2 Renormalons vs. the operator product expansion

In this section we will try to elucidate which properties of renormalons are shared by the non-perturbative conributions. As a test case we choose the total cross section of e^+e^- annihilation since in this case a more general framework based on the OPE is also available.

The OPE applies in euclidean space-time. The polarization operator in euclidean space, $P(Q^2)$, is related to an integral over the total cross section $\sigma(s)$:

$$P(Q^2) \;=\; \frac{Q^2}{\pi}\int\frac{\sigma(s)}{(s+Q^2)s}. \tag{4}$$

At large energies the total cross section is calculable perturbatively:

$$\sigma(s) \;=\; (parton\ model)\cdot\left(1+\frac{\alpha_s(s)}{\pi}+1.409(\frac{\alpha_s(s)}{\pi})^2-12.805(\frac{\alpha_s(s)}{\pi})^3+...\right)\quad, \tag{5}$$

where the first three terms in the expansion have been calculated explicitly see, for example, the second of reference [1]. Phenomenologically, $P(Q^2)$ can also be analyzed via QCD sum rules [6] which approximate $P(Q^2)$ at moderately large Q^2 as

$$Q^2\frac{dP(Q^2)}{dQ^2} \;\approx\; (parton\ model)\cdot\left(1+\frac{\alpha_s(Q^2)}{\pi}+\frac{\langle\alpha_s(G^a_{\mu\nu})^2\rangle}{Q^4}\frac{\pi}{6}+...\right), \tag{6}$$

where $G^a_{\mu\nu}$ is the gluon field strength tensor and the vacuum expectation value of $(G^a_{\mu\nu})^2$ incorporates non-perturbative effects. The theoretical framework behind (6) is the OPE for the T-product of two electromagnetic currents while the main hypothesis is that the power like corrections are responsible for violation of the asymptotic freedom once one starts to descend from very high Q^2 and to approach $Q^2 \sim 1\ GeV^2$. The cross section which corresponds literally to (6) coincides with (5) as far as α_s correction is concerned and a term proportional to $\delta'(Q^2)$.

Now, the renormalons fall so to say in between the two representations (5) and (6) where the former apparently does not contain power like corrections while the latter emphasizes the Q^{-4} terms. Indeed, on one hand, renormalons are particular perturbative graphs with n insertions of the vacuum polarization into a gluon line. Therefore, they are included into the expansion (5). For large n and a one-term β-function the renormalon contribution is

proportional to

$$P(Q^2)_{renorm} \sim \alpha_s^n(Q^2)\frac{1}{Q^4}\int_0^{\sim Q^2} d^4k \left(ln\frac{Q^2}{k^2}\right)^n b_0^n \sim \alpha_s^n(Q^2)n!(\frac{b_0}{2})^n \tag{7}$$

where we have retained only the contribution of the extremum of the integrand at

$$k_{eff}^2 \sim Q^2 exp(-n/2) \tag{8}$$

which is reponsible for the $n!$ growth at large n.

On the other hand, one can argue that the perturbative calculation is of no relevance once $n > N_{cr}$ where the N_{cr} is defined by the condition $(k_{eff}^2)_{N_{cr}} \sim \Lambda_{QCD}^2$. Indeed, once we approach the Landau pole nonperturbative effects are expected to become important. The renormalon contribution at N_{cr} is of order

$$P(Q^2)_{N_{cr}} \sim \frac{\Lambda_{QCD}^4}{Q^4} \tag{9}$$

and exhibits in this way the power like corrections, shown in eq. (6). These two facets of renormalons reveal mixing of perturbative and non-perturbative contributions so that only their sum is uniquely determined, as is emphasized in Refs [18, 19] (for discussion see also [6, 20, 21]).

Imagine now that we would like to build up a phenomenology of the power like corrections based on renormalons. A straightforward logical way would be to evaluate all the perturbative terms until they start to blow up because of the $n!$ behaviour (7); define in some way the perturbative series and postulate the existence of non-perturbative terms which would compensate for the arbitrariness in defining the perturbative part. From the practical point of view, such a program would be difficult to implement since, for example, one of he most advanced calculations of the perturbative expansion (5) does not reproduce yet the $n!$ behaviour. In principle, such a procedure would reproduce the Q^{-4} behaviour of the non-perturbative contribution. The Q^{-4} corrections would be subordinated however to many terms in the perturbative expansion and hence insignificant. The relations among the Q^{-4} corrections in various channels would be procedure dependent.

Instead, one can pick up the renormalons from the very beginning by rewriting one-gluon exchange with running coupling constant $\alpha_s(k^2)$ where k is the momentum flowing through

the gluon line, as

$$\frac{1}{Q^4}\int_0^{\sim Q} d^4k\alpha_s(k^2) \;=\; \frac{1}{Q^4}\int d\sigma\int d^4k\left(\frac{k}{\Lambda_{QCD}}\right)^{-2\sigma b_0} \;\sim\; \left(\frac{\Lambda_{QCD}}{Q}\right)^4\int\frac{d\sigma}{4-2\sigma b_0} \qquad (10)$$

and defining the integral over the pole at $\sigma = 2/b_0$ as, say, its principal value. Such a procedure corresponds to using the expansion (6) along with representing the matrix element $< \alpha_s(G^a_{\mu\nu})^2 >$ by its perturbative part [21]:

$$\langle\alpha_s(G^a_{\mu\nu})^2\rangle_{pert} \;=\; \frac{3}{2\pi^2}\Lambda^4_{QCD}\int\frac{d\sigma}{1-\sigma b_0/2}. \qquad (11)$$

Note that the OPE can be used for the evaluation of the renormalon graphs since the momentum flowing through the line is much smaller than Q^2, see eq. (8). Calculating, furthermore, the renormalon contribution to various crosssections one would reproduce the relations among the Q^{-4} corrections following from the OPE since the substitution of a particular value of the matrix element (11) does not affect the general relations.

Our next hypothesis would be that non-perturbative contributions are proportional to those of renormalons. The reason is that the both are associated with large distances of order Λ^{-1}_{QCD} and therefore factorize simultaneously. In the case of the total cross section this assumption is obviously true since both the soft-perturbative and non-perturbative effects are absorbed into the one and the same matrix element $< (G^a_{\mu\nu})^2 >$. Finally, the whole machinery of the sum rules would be reproduced if one assumes that the non-perturbative effects enhance the renormalon contributions so that

$$\langle(G^a_{\mu\nu})^2\rangle_{pert} \ll \;\langle(G^a_{\mu\nu})^2\rangle_{non-pert}. \qquad (12)$$

The inequality (12) is understood here in the practical sense that one may neglect at moderately large Q^2 high orders in $\alpha_s(Q^2)$ as compared to the Q^{-4} terms. This assumption does not have a sound theoretical foundation as yet and should be considered as a heuristic one, or as motivated by the analysis of the data. It is crucial, however, to build up a phenomenology starting from renormalons [21].

Let us also mention two other points concerning renormalons. First, we took it for granted that α_s in eq. (10) depends on the momentum flowing through the gluon line. This is in

fact not obvious and should have been substantiated by a careful analysis. This is a difficult point of the renormalon technique. Second, renormalons are pure perturbative constructs and they cannot distinguish which hadronic matrix element of $(G^a_{\mu\nu})^2$ is considered. The enhancement due to the non-perturbative effects can depend on the hadronic state, on the other hand. Consideration of total cross sections is related to the vacuum epectation value of $(G^a_{\mu\nu})^2$.

To summarize, the phenomenological success of the QCD sum rules suggests the validity of two basic observations concerning non-perturbative terms, namely, that (a) non-perturbative terms share the factorization properties exhibited by renormalons and (b) non-perturbative contributions are large in the sense that one can match the power-like corrections with the first one or two terms of perturbative expansions.

In the subsequent sections we will try a similar kind of phenomenology on the $1/Q$ corrections to the shape variables, in which case there is no OPE but the renormalon technique is still available.

3 Universality of the $1/Q$ corrections to event shapes.

In this section we summarize and further explore the consequences of the universality of the $1/Q$ corrections to event shape variables.

In [11] we argued for the universality of the $1/Q$ corrections based on the following observations:

(a) To all orders in perturbation theory, the $1/Q$ corrections in the renormalon technique come from soft gluons.

(b) The soft gluons factorize into a universal factor in the above. Namely, for an observable O we can write:

$$\langle O\rangle_{1/Q} \;=\; R_O(\alpha_s(Q^2))E_{soft}. \tag{13}$$

Thus, R_O are O dependant and perturbatively calculable whereas E_{soft} are universal and are given by

$$E_{soft} \;=\; \frac{1}{Q}\int\frac{dk_\perp^2}{k_\perp^2}\gamma_{eik}(\alpha_s(k_\perp))k_\perp \tag{14}$$

where γ_{eik} has been calculated in perturbation theory up to the two loop level [25, 26]. It has also been noted that the same quantity appears in the radiative corrections to many hard processes [27, 28, 26]

(c) Combining (a) and (b), together with the discussion of sect. 2, we argue that since we are looking at the same e^+e^- annihilation process the above is true non-peturbatively as well.

At this point we would like to qualify the above remarks: The considerations of this paper as well as the earlier one [11], apply in the kinematic domain near the semi-inclusive 2 jet limit; i.e. $T \to 1$. $1/Q$ corrections, in fact, do not just arise from this domain but also from the multi-jet kinematic region. Eq (13) is a statement about such corrections as well and emphasizes that multi-jet configurations are associated with powers of $\alpha_s(Q^2)$. The region $T \to 1$ itself will be sudakov suppressed at high enough energies and for such kinematic regimes the multi-jet configurations are important. In the last section of this paper we return to a discussion of this point. For the moment, however, we restrict ourselves only to the region where the $1/Q$ corrections near the 2-jet limit are important.

We shall estimate the numerical value of QE_{soft}, including non-perturbative terms, in the next section and identify the corresponding non-perturbative quantity in terms of jet masses in sect. 5. In the remainder of this section we will discuss further physical quantities , observed in e^+e^- annihilation which also receive $1/Q$ corrections and calculate the corresponding R_O.

We turn first to a discussion of the jet opening angle. The energy weighted jet opening angle is an infrared safe quantity and is defined as [22]:

$$\langle sin^2\eta\rangle \;=\; \langle\frac{\Sigma E_i sin^2\delta_i}{Q}\rangle \tag{15}$$

where E_i and δ_i are the energy and the angle with respect to the jet axis of i-th particle, respectively. Here we would like to demonstrate the existence of $1/Q$ corrections to this observable using the renormalon technique.

Consider the contribution to this quantity from the gluon emission process the crossection for which is:(k is the gluon momentum and p_1 and p_2 those of the quark and antiquark

respectively)

$$\frac{1}{\sigma_0}\frac{d\sigma}{dx_1dx_2} \;=\; \frac{C_F}{2\pi}\alpha_s\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)} \tag{16}$$

where $x_i = 2E_i/Q$. Let us change variables from x_1, x_2 to x_2 and $x = k_\perp/Q$ More explicitly:

$$x \;=\; \frac{\sqrt{(1-x_1)(1-x_2)(x_1+x_2-1)}}{x_2} \tag{17}$$

supposing that the gluon is emitted into the same hemisphere as the antiquark with energy E_2. Then:

$$\frac{1}{\sigma_0}\frac{d\sigma}{dx_2dx} \;=\; \frac{C_F}{\pi}\alpha_s\frac{(x_1^2+x_2^2)(x_1+x_2-1)}{(1-x_2)(x_2-2(1-x_1))}\frac{1}{x} \tag{18}$$

with

$$x_1 \;=\; 1-\frac{x_2}{2}+\frac{x_2}{2}\sqrt{1-\frac{4x^2}{1-x_2}} \tag{19}$$

and,

$$0 \leq \; x \; \leq \frac{\sqrt{1-x_2}}{2}. \tag{20}$$

Anticipating that the $1/Q$ correction comes from the region of soft gluons $((x_2, x_1 \sim 1, x_3 \to 0)$ we see that for gluons $Esin^2\delta$ is $O(\omega)$ whereas for the energetic quantities it is $O(\omega^2/Q)$. Hence we keep only the contribution of gluons as far as the $1/Q$ terms are concerned.

Thus,

$$\langle\frac{Esin^2\delta}{Q}\rangle \;=\; \int_0^1 dx_2\int_0^{1/2\sqrt{1-x_2}}\frac{dx}{x}\frac{2C_F}{\pi}\alpha_s\frac{x^2(x_1^2+x_2^2)(x_1+x_2-1)}{(1-x_2)(x_2-2(1-x_1))(2-x_1-x_2)}. \tag{21}$$

Now to get the renormalon contribution, we led $\alpha_s \to \alpha_s(k_\perp^2)$ [23] and consider the limit $x_1, x_2 \sim 1$ and x small. Then:

$$\langle\frac{Esin^2\delta}{Q}\rangle \;\approx\; \frac{4C_F}{\pi}\int_0^1 dx_2\int_0^{\frac{1}{2}\sqrt{1-x_2}} dx\cdot x\frac{\alpha_s(k_\perp^2)}{(1-x_2)(1-\frac{x_2}{2}-\frac{x_2}{2}\sqrt{1-\frac{4x^2}{1-x_2}}}. \tag{22}$$

Interchanging the order of integration we get:

$$\langle\frac{Esin^2\delta}{Q}\rangle \;\approx\; \int_0^1 dx\cdot x\int_0^{1-4x^2}dx_2\frac{4C_F}{\pi}\frac{\alpha_s(k_\perp^2)}{(1-x_2)^2+x^2} \;\approx\; \frac{\pi}{2Q}\int_0^{Q/2}dk_\perp\cdot\frac{4C_F}{\pi}\alpha_s(k_\perp). \tag{23}$$

The general features at the origin of these $1/Q$ corrections are the same as discussed in the previous paper [11], i.e. they come from a region of soft gluons and close to the 2-jet limit.

As final example of the $1/Q$ corrections to jet properties let us consider the expectation value of the fractional energy $\eta = 2E(\delta)/Q$ emitted inside a double cone of half opening angle δ centered around the thrust axis. This too is an infrared safe quantity [24]. We consider the n-th moment of η:

$$M^n(\delta) \;=\; \int \eta^n \rho(\eta) d\eta \tag{24}$$

where $\rho(\eta)$ is the probability density describing the emission of the fractional energy η inside the cone.

We write this moment as [4]:

$$M^n(\delta) \;=\; 1 - \int (1-\eta^n)\rho(\eta) d\eta \tag{25}$$

where the integration region is one with at least one parton outside the cone. Thus for a $q\bar{q}g$ final state:

$$M^n(\delta) \;=\; 1 - \frac{C_F}{2\pi}\alpha_s \int (1-\eta^n) \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} dx_1 dx_2. \tag{26}$$

We are interested in determining if there are any power corrections linear in Q^{-1} to this quantity. For this, we again concentrate on the region of soft gluons. We note that in the above the integration region is subject to the following constraints for the gluon angle and energy:

$$\eta \;=\; 1 \; if \; sin\theta < sin\delta; \quad \eta \;=\; 1 - x_3 \; if \; sin\theta > sin\delta. \tag{27}$$

For soft gluons we expand $(1-\eta^n) \;\approx\; n x_3$ and thus we are let to consider:

$$-\frac{nC_F}{2\pi}\alpha_s \int dx_2 dx_1 \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} x_3$$

subject to the constraint (27). In the soft gluon region this becomes in terms of the variables x_2, x:

$$\langle M^n(\delta) \rangle_{1/Q} \;=\; -n\frac{2C_F}{\pi} \int dx_2 \int \frac{dx}{x} \frac{\alpha_s(k_\perp^2)}{1-x_2} \{1 - x_2 + \frac{x^2}{1-x_2}\}. \tag{28}$$

The integration region is determined from:

$$\frac{1}{2}(1-x_2)sin\delta \;\leq\; x \;\leq \frac{1}{2}\sqrt{1-x_2}; \quad 0 \leq\; x_2 \;\leq 1. \tag{29}$$

We next interchange the order of integration and concentrate on the small x region to extract consistently the $1/Q$ term using the renormalon technique. We get:

$$\begin{aligned}\langle M^n(\delta)\rangle_{1/Q} &\approx -n\frac{2C_F}{\pi}\int_0^{\frac{1}{2}sin\delta}\frac{dx}{x}\int_{1-\frac{2x}{sin\delta}}^{1-4x^2}\alpha_s(k_\perp^2)\frac{1}{1-x_2}\{1-x_2+\frac{x^2}{1-x_2}\} \\ &= -n\frac{4C_F}{\pi}\frac{1}{Qsin\delta}\int_0^{\frac{Q}{2}sin\delta}dk_\perp\alpha_s(k_\perp^2)-n\frac{2C_F}{\pi}\frac{sin\delta}{Q}\int_0^{\frac{Q}{2}sin\delta}dk_\perp\alpha_s(k_\perp^2).\end{aligned} \tag{30}$$

The conditions for the applicability of this calculation is:

$$\frac{n\Lambda_{QCD}}{Q\cdot sin\delta} \ll 1. \tag{31}$$

Thus we expect non-perturbative corrections to the quantity $M^n(\delta)$ which for small δ goes like $n/Q\delta$. Corrections of the form $1/Q\delta$ for small δ in jet processes was discusssed in [9].

We should again emphasize here that in order to compare these predictions with the data one has to make sure that the kinematic region under study is such that the 2-jet configurations are not strongly sudakov suppressed.

4 Comparison with experimental data.

In this section we compare the theoretical predictions based on the universality of the $1/Q$ terms with experimental data.

At this point, we have four observables $\langle 1-T\rangle, \langle C\rangle, \langle\sigma_L\rangle$ and $\langle Esin^2\delta/Q\rangle$ on which the idea of the universality of the $1/Q$ corrections can be tested experimentally. The results are shown in Table 1 below. In listing the predictions we have used the following observation consistent with our approach to renormalon phenomenology discussed in section 2. In calculating $< 1-T >$, we proceed as in [11] and obtain the corresponding $1/Q$ term in the lowest non trivial order. To this order this is the same as the $1/Q$ correction to the heavy jet mass, M_h^2. Since the effect comes from the soft region where the coupling is order unit, we must resum all such contributions. In higher orders, the light jet massM_l^2 will similarly give a non vanishing $1/Q$ correction. Thus we must include this in our non perturbative estimate of $< 1-T >$ as well. Consistent with our discussion in section 2 we must therefore allow for a non perturbative enhancement factor of 2. This is taken into account in our predictions

for the non perturbative contributions to $< 1 - T >$ as well as to $< C >$, and to$< \sigma_L/\sigma_T >$. We shall come back to a discussion of this point in sections 4 and 5.

Table 1

Observable	α_s *expansion*	$1/Q$ *correction* (GeV)	R_o	QE_{soft} (GeV)
$\langle 1-T \rangle$	$0.335\alpha + 1.02\alpha^2$	~ 1	2	~ 0.5
$\langle C \rangle$	$1.374\alpha_s + 4\alpha_s^2$	~ 5	3π	~ 0.53
$\langle \frac{\sigma_L}{\sigma_T} \rangle$	α_s/π	~ 0.8	$\frac{\pi}{2}$	~ 0.51
$\langle \frac{E sin^2\delta}{Q} \rangle$	$2\alpha_s/\pi$	1.20 ± 0.25	π	0.38 ± 0.08

The estimate of the $1/Q$ correction for the first three observables is taken from [8]. This paper contains also comparison of theoretical predictions, in a different form, with experimental data treating one of variables as an input. In the next section we shall be able to fix the overall scale as well. Data on the energy weighted jet angle is from [4]. The energy range for the last estimate is also different from the first three: 8-30 GeV for the latter compared to upto LEP energies for the former. We should note that using the results of [4] for the $1/Q$ correction to thrust, we would find for the quantity $QE_{soft} = 0.30 \pm 0.08$.

We did not list prediction (31) because it cannot be confronted directly with experimental fits to $1/Q$ corrections. However, the data on the moments exist for various values of δ, n [4]. An inspection of the corresponding experimental curves reveals that the data do not show large non perturbtaive corrections for small δ as predicted in (31). Taken at face value, the data are the most serious disagreement with theory that we find. Updating of the analysis of the data appears desirable.

5 The overall scale of the $1/Q$ corrections.

In this section we identify the non perturbative scale associated with the $1/Q$ corrections in e^+e^- annihilation, i.e we infer the non perturbative counterpart of QE_{soft}. In order

to motivate this identification, we briefly recall the discussion of thrust from [11]. There it was shown that to all orders the $1/Q$ corrections come from soft gluons and from the neighbourhood of the two jet limit. In particular, we identified from simple kinematics and from the infrared safety of T, that to all orders, as far as the $1/Q$ corrections are concerned:

$$\langle 1 - T \rangle_{1/Q} = \frac{M_1^2}{Q^2} + \frac{M_2^2}{Q^2} \tag{32}$$

Here we consider two hemispheres divided by a plane perpendicular to the thrust axis and M_1^2 and M_2^2 are the invariant masses in these. In the limit of soft gluons $M_i = \sum_j 2p_i k_j$, with k the momenta of the gluons.Thus in perturbation theory, $\langle 1-T \rangle_{1/Q}$ is essentially twice the jet mass squared. By using the factorization of soft gluons, we have upto corrections of $O(\alpha_s(Q^2))$:

$$\langle 1 - T \rangle_{1/Q} = 2E_{soft} \tag{33}$$

with,

$$QE_{soft}|_{renormalon} = \int \frac{dk_\perp^2}{k_\perp^2} \gamma_{eik}(\alpha(k_\perp^2))k_\perp. \tag{34}$$

where, in the integrand we pick up the contribution of the renormalon pole in α_s.

It might worth emphasizing that, once it is established that the $1/Q$ corrections are associated with soft gluons the relation (33) between the jet mass and $< 1 - T >$ becomes purely kinematical. It is inherent to hadronization models (for review see [4, 5]), or to soft perturbative contributions (see, e.g., [29]).

Now, we use this kinematical nature of relation (33) to fix the overall scale of non-perturbative contribution to the $1/Q$ terms. Namely, we argued in [11] and in the earlier sections that the relation which are true to all orders in α_s normalized at low mass scale of $\sim \Lambda_{QCD}$ should be true non perturbatively as well. Thus, we are naturally led to the non perturbative identification of $Q^2 E_{soft}$, near the 2-jet limit, as the average non perturbative correction to the square of the jet masses. Phenomenologically the hadronization correction to the jet masses is parametrized thus:

$$\langle M_{had}^2 \rangle = \lambda Q \tag{35}$$

which corresponds to jet momenta receiving a negative correction of order λ. For example, in the "tube" model of hadronization one finds (see [5] for a review),

$$\lambda = \int d^2p_\perp \rho(p_\perp) p_\perp \tag{36}$$

where $\rho(p_\perp)$ gives the $p_\perp$ distribution of hadrons in a rapidity-$p_\perp$ "tube". In particular, according to the analysis quoted in [5], from the experimental data on jet masses, one finds:

$$\lambda \sim 0.5 \; Gev. \tag{37}$$

In fact in simple physical models one can directly see the non perturbative connection between the $1/Q$ corrections to the observables and to the quantity λ. Consider for instance the energy weighted jet opening angle $\langle sin^2\eta\rangle$. Let $\tilde{\rho}(z, p_\perp)$ denote the appropriately normalized distribution of hadrons in a jet with longitudinal momentum fraction z and perpendicular component $p_\perp$. Thus, if p_3 is the component of the hadron momenta along the jet axis, then $z = 2p_3/Q$. Whence,

$$\langle sin^2\eta\rangle = \int_0^1 dz \int d^2p_\perp \tilde{\rho}(z, p_\perp) \frac{p_\perp^2}{1/4z^2Q^2 + p_\perp^2} \tag{38}$$

The $1/Q$ terms can only come from the neighbourhood of $z = 0$, hence:

$$\langle sin^2\eta\rangle = \int d^2p_\perp \rho(p_\perp) \int_0^1 dz \frac{p_\perp^2}{1/4z^2Q^2 + p_\perp^2} \tag{39}$$

where, $\tilde{\rho}(0, p_\perp) = \rho(p_\perp)$. Thus,

$$\langle sin^2\eta\rangle \;=\; \pi\frac{1}{Q}\int d^2p_\perp \rho(p_\perp) p_\perp \;=\; \pi\frac{\lambda}{Q} \tag{40}$$

The closeness of the two approaches is hard to miss. The reason for this similarity is that both renormalons and the standard hadronization picture identify bounded intrinsic $p_\perp$ as a manifestation of the non perturbative effects. The corresponding derivations of the $1/Q$ corrections therefore become identical in this approximation, with the replacement $\gamma_{eik}/k_\perp^2 \to \pi\rho$.(Compare equations (22) and (39)).

As a final example consider the moments $M^n(\delta)$ discussed in section 3. The $(sin\delta)^{-1}$ dependance is easily understood in the hadronization picture. Indeed, hadrons with energies

upto $p_\perp/sin\delta$ leave the cone with opening angle δ and reduce the flow of energy inside this cone, in agreement with the renormalon picture given in section 3. (See equation (30)).

To summarize, we have proposed to identify the non perturbative scale from pure kinemtical considerations and the theoretical input on the dominant contribution of soft gluons to $1/Q$ corrections.So far the absolute scale of non-perturbative terms was fixed only within the operator product expansion in the euclidean region [6].The generalization of this idea to Minkowskian space has thus far not produced a handle on the phenomenology of $1/Q$ corrections since the operators become nonlocal [9, 12]. It is amusing that just the same transition from euclidean to minkowskian space which is responsible for the change of local operators into nonlocal ones brings new tools based on the minkowskian kinematics. However, as emphasized earlier,our identification can be correct only to order $O(\alpha_s(Q^2))$. In the absence of the OPE the generalization to higher orders in $\alpha_s(Q^2)$ is not straightforward at all, as we discuss in the next section.

6 Comparison with other methods. Conclusions.

In this paper we have attempted to bridge the renormalon picture of $1/Q$ corrections with the phenomenological hadronization models. The considerations of this and the previous paper [11] may be thought of as providing a field theoretic argument for the justification, near the 2-jet limit, of these hadronization models which were so far considered at a purely phenomenological level. Conversely, one may turn the argument around and use the success of the phenomenological models to justify the picture of renormalon inspired phenomenology discussed in section 2.

It might be worth mentioning that not all renormalon-based approaches end up with the same kind of phenomenology as outlined above [1]. Indeed, consider models with (fictitious) gluon mass μ [8] or with freezing of the coupling constant at a scale μ_I [10]. The results based on this model were reviewed recently in [30]. It is no surprise that many predictions coincide with what we have. A more detailed consideration unveils however that this coincidence is rather formal. The easiest way to appreciate the difference is to consider predictions for the

[1] The following discussion is added following the suggestion of a referee report.

difference of masses of a heavy and light jets. According to [8, 10, 30]:

$$\langle\frac{M_h^2 - M_l^2}{Q^2}\rangle_{1/Q} \;=\; \langle\frac{M_h^2}{Q^2}\rangle_{1/Q} \;=\; \langle 1-T\rangle_{1/Q} \;=\; \frac{16\alpha_s}{3\pi}\frac{\mu}{Q} \;=\; \frac{16}{3\pi}\frac{\mu_I}{Q}\bar{\alpha}_0(\mu_I) \tag{41}$$

Note that the prediction for $\langle 1-T\rangle_{1/Q}$ coincides with (33) upon identification of our QE_{soft} with $(8/3\pi)\mu_I\bar{\alpha}_0(\mu_I)$. This identification works in some other cases as well. However, the central point point about eq (41) is that adding or subtracting M_l^2 does not change anything. This($M_l^2 = 0$) is inherent to models with a gluon mass or with the frozen coupling [30]. The reason is that these models generalize to non-perturbative physics basing on one-loop calculations.

On the other hand, within the approach developed in this paper we reproduce in the leading approximation the standard hadronozation picture as described, say, by the tube model. Therefore, the $(M_h^2 - M_l^2)$ difference depends on the distribution of nonperturbative momenta and vanishes if this distribution is dominated by a characteristic value. It has been known for some time [4] that this picture is consistent experimentally:

$$\langle\frac{M_h^2 - M_l^2}{Q^2}\rangle_{1/Q} \ll \langle\frac{M_h^2}{Q^2}\rangle_{1/Q}. \tag{42}$$

Similarly, our prediction for the $1/Q$ corrections reads as

$$\langle 1-T\rangle_{1/Q} \;=\; \langle\frac{M_h^2 + M_l^2}{Q^2}\rangle_{1/Q} \;\approx\; 2\cdot\langle\frac{M_h^2}{Q^2}\rangle_{1/Q} \tag{43}$$

This difference of a factor of 2 compared to (41) is not immediately manifest because of arbitrariness in normalization of μ_I.

One might wonder how it is possible to have different pictures within the one and the same renormalon approach. The point is that, as emphasized in sect. 2, the enhancement factor for nonperturbative effects defies naive analysis of divergencies of perturbative expansions. In particular, the model with gluon mass makes non perturbative effects, as imitated by non analytical terms in mass squared, subordinate to the perturbative contributions in the most transparent way. Indeed, since the μ/Q correction is associated with a small fraction of the phase space this term is always a minor effect on the background of the ordinary perturbative contribution. Within this model, it is inconsistent to consider both M_h and M_l

as nonvanishing unless at least α_s^2 corrections are included. In fact, more careful analysis would reveal that even higher orders are needed for consistent treatment of $1/Q$ terms.

Within the aproach which allows for enhancement of non perturbative effects there is no difficulty to retain $1/Q$ terms even when the perturbative terms are small numerically. Similarly, in QCD sum rules the condensate terms dominate over pure perturbative contributions at moderate Q^2 (see sect. 2). It might be worth noting that according to Ref [30] the corrections to the prediction of the model with the frozen coupling can be as large as 50%. In the renormalon language, our eq (33) corresponds to summation of all these large corrections so that the remaining uncertainty is of order $\alpha_s(Q^2)$ [11, 12]. In a sense this corresponds to summing the soft contributions from the multi-jet configurations as well. However, this summation cannot be done in the model with frozen coupling since the freezing has no clear field theoretical realization.

Let us also mention paper [31] which tests, in particular,the conclusions of references [11, 9] on the example of large n_f. Although an agreement is found in lowest orders evaluated explicitly, the question is raised on possible existence of $\alpha_s(Q^2) \cdot lnQ^2$ corrections in higher orders, due to multijet events. It might be worth emphasizing, therefore, that all the statements on two-jet dominance which are made above are to be qualified for the energy being not so high as to make the effect of the Sudakov suppression appreciable. If not,then terms like $lnQ^2 \cdot \alpha_s(Q^2)$ do appear. Moreover, these terms cannot be dealt directly within the renormalon technique. The way out is well known however. Namely, one considers the resummed cross section and renormalon corrections to it [7, 9, 11, 12]. For example, in case of thrust instead of $< 1 - T >_{1/Q}$ one has to consider [12]

$$\langle e^{-\nu(1-T)} \rangle_{1/Q} \tag{44}$$

where the parameter ν is large enough to ensure the dominance of the two-jet configuration despite the Sudakov suppression. In our discussion of the experimental data we did not introduce ν since the fits to $1/Q$ corrections [8, 10] do not indicate any energy dependence.

Another issue raised in Ref [31] is the search for a proper choice of a shape variable which would be free of $1/Q$ corrections and hence could be used for reliable extractions of the value of α_s. We would like to notice that consideration of spherocity could be promising from this

point of view. The spherocity S is defined as [32]

$$S \;=\; \left(\frac{4}{\pi}\right)^2 \; min\left(\frac{\Sigma_i |p_T^i|}{\Sigma_i |P_i|}\right)^2 . \tag{45}$$

Perturbatively, it is given as [33]

$$\langle S\rangle \;=\; \frac{64}{\pi^2}\frac{\alpha_s}{3\pi}\left(-\frac{229}{9}+64ln\frac{3}{2}\right). \tag{46}$$

Explicit calculation shows that the spherocity is free from $1/Q$ correction in the leading order in α_s. Thus to this accuracy:

$$\langle S\rangle \;=\; \frac{\alpha_s}{3\pi}(3.28) \;+\; O(1/Q^2) \tag{47}$$

Instread of giving details of derivation let us remind the reader of the simple estimate of the non perturbative contribution to the spherocity [33]:

$$\Delta S_{non-pert} \;\approx\; \left(\frac{4}{\pi}\right)^2 \frac{< p_T >^2}{Q^2} < n(Q^2) >^2 \;\;\sim\;\; \frac{lnQ^2}{Q^2} \tag{48}$$

where $n(Q)$ is the multiplicity and $< p_T >$ is the characteristic non perturbative transverse momentum. Since we know that renormalons do reproduce the basic features of the phenomenological hadronization models, it is no surprise that that there is no $1/Q$ renormalon here. Moreover, this kind of argument appears to work for multijet events. Thus, there are good chances that the spherocity is indeed free from $1/Q$ corrections to higher orders in $\alpha_s(Q)$ as well.

As we emphasized above, all of our considerations until now have only dealt with the leading $1/Q$ corrections to the quark jets. For gluon jets, the corresponding anomalous dimension γ_{eik} is different in perturbation theory. Thus it appears likely that a different non perturbative parameter will be needed to describe situations dealing with gluon jets. Continuing in a similar spirit,we recall that our estimates of the $1/Q$ corrections have been carried out in the kinematic region where the dominant contribution comes from the neighbourhood of the 2-jet limit. However it is clear that though suppressed, three jet processes will have to be taken into account in the next order of approximation,i.e order $\alpha_s(Q^2)$. New non perturbative parameters could arise here. It is a challenge to implement such corrections into the developed framework.

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